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Section A

Spin rotators and split Siberian Snakes *

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The study of spin effects in the collision of polarized high energy beams requires flexible and compact spin rotators to manipulate the beam polarization direction. Design criteria and specific examples are presented for high energy, orbit transparent spin rotators ranging from small angle rotators to be used for the excitation of spin resonances to large angle rotators to be used as Siberian Snakes. It is shown that all the requirements for spin rotators can be met with a simple 6-magnet spin rotator design, for which a complete continuous solution is presented.

1. Introduction

The acceleration of polarized protons in circular accelerators requires tools to manipulate the polarization direction in much the same way as magnetic multipoles are used to manipulate the optical properties of the beam. Localized spin rotators distributed around the ring, which are called Siberian Snakes [1], are used to stabilize the polarization during acceleration. Spin rotators are also used around the interaction region of colliding beams to prepare the polarization direction that is needed by the experiments. In high luminosity, high energy proton–proton colliders spin rotators cannot easily be placed immediately next to the collision point, but have to be placed outside of the dipoles that combine the two colliding beams. This requires the preparation of a spin direction that is different from the required polarization direction at the collision point to take the spin precession experienced by the beam in the dipoles into account. Clearly the spin rotator design then needs to allow for a rotation into any horizontal direction since the precession in the dipoles, that combine the beams, depends strongly on the beam energy.

Both the spin direction and the velocity direction of a charged particle are affected by electric and magnetic fields. The change of the velocity direction is due to the Lorentz force:

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{e}{m\gamma}[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}], \tag{1}$$

where I assumed that there is no electric field $\boldsymbol{E}$ parallel to the velocity $\boldsymbol{v}$ and thus no change in the energy of the particle. Eq. 1 can then be rewritten as

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{\Omega}_{\mathrm{c}} \times \boldsymbol{v} = \frac{e}{m\gamma}\left[\boldsymbol{B} + \frac{\gamma^2}{\gamma^2 - 1}\frac{\boldsymbol{E}_{\perp} \times \boldsymbol{v}}{c^2}\right] \times \boldsymbol{v}, \tag{2}$$

where $\boldsymbol{\Omega}_{\mathrm{c}}$ is the cyclotron frequency.

The change of the spin direction, on other hand, is given by the Thomas–BMT equation [2]:

$$\begin{aligned}\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t} &= \boldsymbol{\Omega} \times \boldsymbol{s} \\ &= -\frac{e}{m\gamma}\left[(1 + G\gamma)\boldsymbol{B}_{\perp} + (1 + G)\boldsymbol{B}_{\parallel}\right. \\ &\qquad \left. + \gamma\left(G + \frac{1}{\gamma + 1}\right)\frac{\boldsymbol{E}_{\perp} \times \boldsymbol{v}}{c^2}\right] \times \boldsymbol{s}.\end{aligned} \tag{3}$$

Since the Lorentz and the Thomas–BMT equation exhibit very different dependencies on the electric and magnetic field it is possible to construct field configurations that only rotate the spin and leave the velocity direction unchanged. The simplest configuration consists of a pure longitudinal magnetic field. The spin precession vector is then

$$\Omega_L = -\frac{e}{m\gamma}(1 + G)\boldsymbol{B}_{\parallel}. \tag{4}$$

For low magnetic fields it is also possible to apply a transverse electric field perpendicular to the magnetic field to exactly cancel the Lorentz force.

$$\boldsymbol{E}_{\perp} = -\boldsymbol{v} \times \boldsymbol{B}_{\perp}. \tag{5}$$

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For such a field configuration the spin precession vector is

$$\Omega_W = -\frac{e}{m\gamma^2}(1+G)\boldsymbol{B}_\perp . \tag{6}$$

Clearly, for high energy particles ($\gamma \gg 1$) it is not possible to find a single magnetic or electric field direction that rotates the spin vector by an angle significantly different from zero since both Ω_L and Ω_W vanish at high energy. However, a sequence of transverse magnetic fields alternating between vertical and horizontal can be found such that the final velocity direction is unchanged and the spin direction is rotated [3]. Note that for pure transverse fields the spin rotation angle is $G\gamma$ times bigger than the orbit deflection angle in a reference frame which rotates with the velocity direction. This means that for high energy particles sizable spin rotations can be achieved with only small orbit deflections. For small deflection angles the rotations commute with each other and the requirement of unchanged orbit can be fulfilled by simply requiring the horizontal and vertical fields to add up to zero, respectively.

The simplest configuration of dipole magnets with zero total deflection and also no orbit displacement consists of six dipole magnets in the following two arrangements:

$$R_V = R(\psi_H)R(\psi_V)R(-2\psi_H)R(-2\psi_V)R(\psi_H)R(\psi_V),$$
$$R_H = R(\psi_V)R(\psi_H)R(-2\psi_V)R(-2\psi_H)R(\psi_V)R(\psi_H), \tag{7}$$

where ψ_H and ψ_V are the spin rotation angles in the horizontal and vertical dipole magnets, respectively, and R stands for the rotation operations. I will use the spinor notation as summarized in the Appendix 1 of Montague [4] to represent the spin rotations:

$$R(\psi_H) = \cos\frac{\psi_H}{2} - i\sigma_3 \sin\frac{\psi_H}{2},$$
$$R(\psi_V) = \cos\frac{\psi_V}{2} - i\sigma_2 \sin\frac{\psi_V}{2}. \tag{8}$$

I will use the subscripts 1, 2, and 3 to refer to the directions along the beam, radial outward, and vertical to the beam, respectively. The overall spin rotation of the two dipole configurations listed above is then given by:

$$\begin{aligned} R_{\binom{H}{V}} = {} & \left[1 - 2\sin^2\frac{\psi_H}{2}\sin^2\frac{\psi_V}{2} - \sin^2\psi_H \sin^2\psi_V\right] \\ & \pm i\sigma_1\left[\tfrac{1}{2}\sin\psi_H \sin\psi_V\left(3 - 8\sin^2\frac{\psi_H}{2}\sin^2\frac{\psi_V}{2}\right)\right] \\ & + i\sigma_2\left[\sin\psi_V\left(2\sin^2\psi_H\sin^2\frac{\psi_V}{2} - \sin^2\frac{\psi_H}{2}\right)\right] \\ & - i\sigma_3\left[\sin\psi_H\left(2\sin^2\psi_V\sin^2\frac{\psi_H}{2} - \sin^2\frac{\psi_V}{2}\right)\right]. \end{aligned} \tag{9}$$

2. Small angle spin rotators

For small angles ψ_H and ψ_V, ignoring any quadratic or higher order terms, the rotation becomes

$$\begin{aligned} R_{\binom{H}{V}} & \approx 1 - \tfrac{9}{8}\psi_H\psi_V \pm i\sigma_1\tfrac{3}{2}\psi_H\psi_V \\ & \approx \cos(3\psi_H\psi_V) \pm i\sigma_1 \sin(3\psi_H\psi_V), \end{aligned} \tag{10}$$

which therefore corresponds to a rotation by the angle $\psi_S = 3\psi_H\psi_V$ around the longitudinal direction of the beam.

Small angle spin rotators with a horizontal rotation axis can be useful to drive spin resonances in the presence of the vertical holding field of circular accelerators [5]. With constant field magnets the resonance frequencies are harmonics of the revolution frequency:

$$f_{res} = n \times f_{rev}. \tag{11}$$

Other resonance frequencies can be obtained by modulating all the vertical bending magnets but keeping the field in the horizontal bending magnets constant. The spin rotation angle is then modulated with the same frequency as the vertical bending magnets. This allows to drive spin resonances at any frequency and manipulate the spin direction using well established nuclear magnetic resonance (NMR) techniques. Note that the spin rotation angle is independent of the beam energy and therefore such a spin rotator can be used even at high energy accelerators. Modulated spin rotators have been constructed using solenoids [6]. However, their usefulness is limited to low energy beams since the obtainable spin rotation angle decreases with increasing beam energy.

Typically the modulated magnetic field is much weaker than the constant fields. For small ψ_V

$$\begin{aligned} R_{\binom{H}{V}} & \approx 1 - \psi_V^2\left[\frac{1}{2}\sin^2\frac{\psi_H}{2} + \sin^2\psi_H\right] \\ & \qquad \pm i\sigma_1\tfrac{3}{2}\psi_V \sin\psi_H - i\sigma_2\psi_V \sin^2\frac{\psi_H}{2} \end{aligned} \tag{12}$$

and the spin rotation angle is

$$\psi_S = \pm 2\psi_V\sqrt{\sin^2\frac{\psi_H}{2} + 2\sin^2\psi_H}. \tag{13}$$

The angle ϕ_S between the rotation axis and the beam direction is given by:

$$\tan\phi_S = \mp\frac{1}{3}\tan\frac{\psi_H}{2}. \tag{14}$$

ψ_S is maximized for $\cos\psi_H = -\frac{1}{8}$ or $\psi_H = 97.2°$. For this value of ψ_H, $\psi_S = 3.2\psi_V$ and $\phi_S = \mp 20.7°$. If the field of the horizontal bending magnets is modulated and the vertical bending magnets are kept constant the overall rotation axis would no longer lie in the horizontal plane.

3. Large angle spin rotators

Typically spin rotators are used to rotate the polarization of a circulating beam into the horizontal plane for scattering experiments in collisions with a counter rotating beam. After the collision the polarization has to be restored to the original polarization direction. The stable spin direction in a circular accelerator is usually vertical which means that the spin rotator has to rotate the polarization from vertical to horizontal.

The string of six dipole magnets described above allows for spin rotation into the horizontal plane which is, at high enough energy, independent of the beam energy. The requirement for rotation from vertical to horizontal can be written as

$$\mathrm{Tr}(\sigma_3 R \sigma_3 R^+) = 0. \tag{15}$$

This results in the following equation for ψ_H and ψ_V:

$$(1 - 4\sin^2\psi_H \sin^2\psi_V) \times \left(1 - 2\sin^2\frac{\psi_H}{2}\sin^2\psi_V\right) = 0. \tag{16}$$

The loci of the two possible solutions are shown in Fig. 1. The first factor is shown as a solid line, the second as a dashed line. Clearly the first factor has solutions with considerably smaller values for the spin rotations in the dipole magnets and therefore requires smaller magnets.

Fieguth [7] described discrete solutions for these 6-magnet spin rotators which he called 'serpents' and classified them using discrete symmetry groups. His two solutions $\psi_H = \psi_V = 45°$ and $\psi_H = \psi_V = 90°$ lie respectively on the two solutions of Eq. (16). However, Eq. (16) shows that there are an infinite number of solutions and that they can be classified into two unconnected solution loci.

Next we insert the condition for the first solution ($\sin^2\psi_H \sin^2\psi_V = \frac{1}{4}$) into the general rotation matrix:

$$\begin{aligned} R_{\binom{H}{V}} &= \left[\tfrac{3}{4} - 2\sin^2\frac{\psi_H}{2}\sin^2\frac{\psi_V}{2}\right](1 \pm i\sigma_1) \\ &\quad + \left[\sin\psi_H \sin^2\frac{\psi_V}{2} - \sin\psi_V \sin^2\frac{\psi_H}{2}\right] \\ &\quad \times (i\sigma_2 + i\sigma_3) \\ &= A(1 \pm i\sigma_1) + B(i\sigma_2 + i\sigma_3). \end{aligned} \tag{17}$$

The polarization direction in the horizontal plane after the spin rotator can then be obtained from the appropriate traces of the rotated spinor $R\sigma_3 R^+$. Note that the rotated spinors show a very different behavior for R_H than for R_V:

$$\begin{aligned} R_H \sigma_3 R_H^+ &= \sigma_2, \\ R_V \sigma_3 R_V^+ &= 4AB\sigma_1 - 2(A^2 - B^2)\sigma_2. \end{aligned} \tag{18}$$

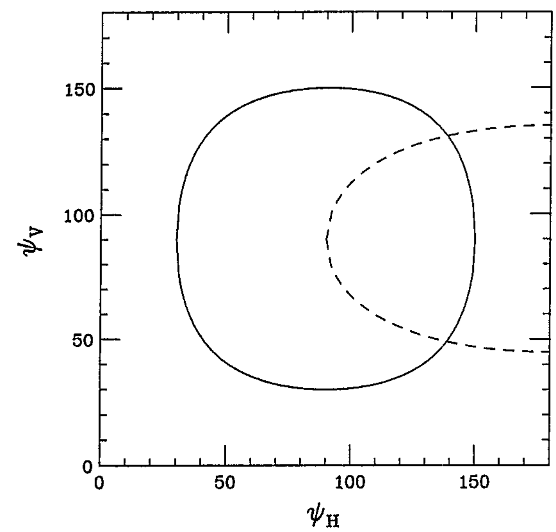

Fig. 1. Loci of the two solutions for the 6-dipole spin rotator that rotates the spin from the vertical to the horizontal direction. ψ_H and ψ_V are the spin rotation angles in the horizontal and vertical bending magnets, respectively.

The spin rotatcr that has a horizontal bending magnet as its first magnet apparently always produces radial polarization whereas the rotator that starts with the vertical bending magnet can produce any polarization direction in the horizontal plane. The angle between the polarization direction and the beam direction for the latter spin rotator is given by:

$$\sin\alpha = 1 - (\sin\psi_H - \sin\psi_V - \sin(\psi_H - \psi_V))^2. \tag{19}$$

Fig. 2 shows this angle as a function of the spin rotation in the horizontal bending magnet.

4. Split Siberian Snakes

In circular accelerators numerous spin resonances are driven by field imperfections and also by the focusing quadrupoles. To avoid depolarization from passing through these spin resonances during acceleration it has been proposed to introduce spin rotators into the circular accelerators [1]. Such spin rotators are called Siberian Snakes. To completely avoid all spin resonances Siberian Snakes have to rotate the spin by 180° around an axes that lies in the horizontal plane. Many designs for Siberian Snakes have been proposed which minimize the beam orbit excursions inside the Snake and allow for varying the direction of the spin rotation axes [8]. For high energy accelerators the Siberian Snakes have to be inserted in pairs with the two Snakes on opposite sides of the ring and with the spin rotation axis perpendicular to each other. It was pointed out by Fieguth [7] and Lee [9] that the spin rotators needed

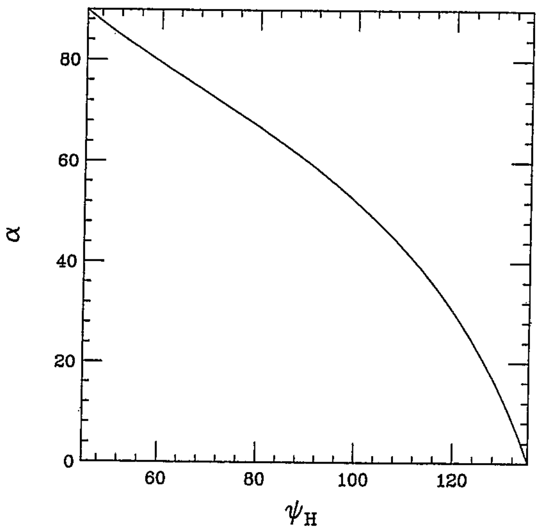

Fig. 2. Angle between the polarization direction and the momentum direction for R_V as a function of the spin rotation angle in the horizontal bending magnet. This also represents the corresponding angle in the middle of a split Snake constructed from spin rotator R_V.

for collider experiments can be used as Siberian Snakes at the same time. The Siberian Snake would then be split into two halves with each half acting as a spin rotator as described above. However, the solutions found so far do not have the required flexibility that allows one to adjust the polarization direction in the center of the split Snake to any direction in the horizontal plane. As will be shown below the 6-magnet spin rotators introduced above can be used to create two classes of split Siberian Snakes: one class that allows for the adjustment of the polarization direction in the center but leaves the overall rotation axes constant and a complementary class that allows for adjustment of the rotation axes but leaves the polarization direction constant.

The requirements for the spin rotation operation of a split Siberian Snake can be summarized as

$$\mathrm{Tr}(\sigma_3 S_2 S_1) = 0 \quad \text{and} \quad \mathrm{Tr}(S_2 S_1) = 0, \tag{20}$$

where S_1 and S_2 are the first and second half of the split Snake, respectively. One can easily verify that

$$S_2 = \sigma_3 S_1^+ \sigma_3, \tag{21}$$

represents a solution to the first requirement. Therefore S_2 can be realized by inverting the order of the magnets and also the sign of the horizontal bending fields of the first half. Inserting the solution into the second requirement gives:

$$\mathrm{Tr}(\sigma_3 S_1^+ \sigma_3 S_1) = 0, \tag{22}$$

which is identical to the requirement that the first half Snake rotates vertical polarization into the horizontal plane. This means any spin rotator that rotates vertical polarization into the horizontal plane can be used as a half Snake. The second half can then be obtained according to the recipe given above.

Next I will examine the relationship between the direction of the horizontal polarization in the middle of the split Snake and the direction of the overall spin rotation axes of the Snake, which also lies in the horizontal plane. The direction of the polarization in the center of the Snake is

$$\boldsymbol{P}^c = \left[\tfrac{1}{2}\mathrm{Tr}(\sigma_1 S_1 \sigma_3 S_1^+), \tfrac{1}{2}\mathrm{Tr}(\sigma_2 S_1 \sigma_3 S_1^+), 0\right] \tag{23}$$

and the spin rotation axes is

$$\boldsymbol{n} = \left[\frac{\mathrm{i}}{2}\mathrm{Tr}(\sigma_1 S_2 S_1), \frac{\mathrm{i}}{2}\mathrm{Tr}(\sigma_2 S_2 S_1), 0\right]$$

$$= \left[\tfrac{1}{2}\mathrm{Tr}(\sigma_2 S_1^+ \sigma_3 S_1), -\tfrac{1}{2}\mathrm{Tr}(\sigma_1 S_1^+ \sigma_3 S_1), 0\right]. \tag{24}$$

Spin rotators consisting of an odd number of dipole magnets can be constructed symmetrically around their midpoint and therefore have the simple property that the conjugated spin rotator can be expressed in terms of the spin rotator itself:

$$R^+ = \sigma_1 R \sigma_1. \tag{25}$$

In this case the polarization in the center of the split Snake and the spin rotation axes are related as:

$$P_1^c = n_2 \quad \text{and} \quad P_2^c = n_1. \tag{26}$$

The spin rotators R_H and R_V, however, consist of an even number of magnets. In this special case the conjugates can be expressed in terms of each other:

$$R^+_{\binom{H}{V}} = \sigma_1 R_{\binom{V}{H}} \sigma_1 \tag{27}$$

and the polarization of the split Snake constructed from one spin rotator determines the spin rotation axes of the Snake constructed from the other and visa versa. For R_H we have

$$\boldsymbol{P}^c = [0, 1, 0] \quad \text{and} \quad \boldsymbol{n} = \left[-2(A^2 - B^2), 4AB, 0\right] \tag{28}$$

and for R_V

$$\boldsymbol{P}^c = \left[4AB, -2(A^2 - B^2), 0\right] \quad \text{and} \quad \boldsymbol{n} = [1, 0, 0], \tag{29}$$

where the expressions A and B were defined in Eq. (17).

With the second split Snake it is therefore possible to adjust the polarization direction in the center of the split Snake without affecting the overall spin rotation axes.

In summary, this paper presents a unified and comprehensive approach to develop all necessary tools for the design of the acceleration and collision of high energy polarized proton beams. These tools were central to the first detailed proposal for 250 GeV on 250 GeV polarized proton collisions at the Brookhaven RHIC [10].

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